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Numerical modeling of local effects on the petroleum reservoir using fixed streamtubes for typical waterflooding schemes

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Abstract. The difficulty of numerical modeling of areal methods of flows redistribution in the oil reservoir is the need for detailed resolution of local hydrodynamic effects and the fine geological structure of the reservoir, which are centimeter-wide, at inter-well distances of the order of several hundred meters. The dimension of computational grids of traditional 3D models of such resolution, even for impact areas containing a small number of injection and production wells, turns out to be excessively large for design calculations. To overcome these limitations, it is proposed to perform a detailed simulation of the flow in two-dimensional cross sections of the reservoir along fixed streamtubes of variable width between each pair of interacting injector and producer wells. Reducing the dimension of the problem allows the use of high-resolution grids to simulate short-term local effects.

In this paper, we present an algorithm for constructing a single fixed streamtube between injector and producer, which provides a minimum error in calculating of flow rate and water cut using a two-phase flow problem of reduced dimension along the streamtube. The algorithm is demonstrated by the example of the two-dimensional two-phase flow problem neglecting capillary and gravitational forces in a homogeneous reservoir of constant thickness for three waterflooding elements corresponding to seven vertical well flooding patterns – standard and inverted four-spot, five-spot and seven-spot, as well as staggered line drive. For these waterflooding elements, efficient streamtubes have been constructed, the relative width of which is approximated by piecewise linear functions. On the example of a staggered line drive or five-spot well patterns, the width of the effective streamtube was parameterized for an arbitrary ratio of the sides of the waterflood element. Presented streamtubes can be used as ready templates for subsequent modeling of geological and technical treatments in the relevant elements of the water flooding of the oil reservoir.

Keywords: oil reservoir, two-phase flow in porous media, geological and technical treatments, fixed stream tube, well patterns, numerical simulation, high-speed models, high-resolution models

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Introduction

Modeling of complex methods of technogenic impact on an oil reservoir, such as oil displacement by polymer solutions, surfactants, acids, hot water, etc. often requires detailed resolution of the geological structure of the reservoir and small-scale hydrodynamic effects that determine the effectiveness of the treatments. The spatial step of such computational grids can reach 0.1–0.01 meters, which is 1–2 orders of magnitude higher than the resolution of traditional three-dimensional numerical hydrodynamical models of the entire reservoir. To describe such effects on the reservoir, it is sufficient to limit the computational domain of the model to a reservoir section containing a small number of interacting wells. Often this is an injection well, through which solutions are injected into the reservoir, and its nearest production wells. However, even local three-dimensional models usually turn out to be unsuitable for multivariate calculations with the required degree of detail due to the excessively large dimension of the computational grid. With a characteristic area of the impact area of about 1 square km and a layer thickness of 10–100 meters, the dimension of the grid with a step about of 0.1 m will reach from 10 to the 10th degree to 10 to the 11th degree of nodes. One of the options for solving the problem can be the use of a flow model with a fixed streamtube (FSTM), which allows at the same time to fundamentally reduce computational complexity and use high-resolution grids (Mazo et al., 2017; Potashev et al., 2016; Shelepov et al., 2016). Such advantages of the model are achieved due to decomposition of a three-dimensional flow problem for

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a series of two-dimensional problems in vertical sections of streamtubes.

We emphasize that the FSTM does not replace the global three-dimensional waterflood dynamics model, but supplements it, since it is intended only to describe relatively short-term local effects in which the general scheme of interaction of production and injection wells in the area is preserved. At the same time, the joint use of high-speed local FSTM with a high-speed super-element model of global waterflooding of the reservoir (Mazo, Bulygin, 2011; Bulygin et al., 2013; Mazo, Potashev, 2020) can fundamentally expand the application area of reservoir simulation design problems by dramatically minimizing computational costs.

The description of flows in oil and water reservoirs using the theory of non-deformable ("rigid", fixed) streamtubes has more than half a century history. In the russian literature, the first statements of the formulations of the general flow problems in non-deformable streamtubes – for an incompressible and compressible single-phase flow, for piston-like displacement of one phase by another and for two-phase flow – can be found in (Krylov et al., 1948; Charny, 1948; Krylov et al., 1962; Charny, 1963). In the english literature as the first works on the application of the streamtube method for petroleum reservoir simulation such papers as (Higgins, Leighton, 1961, 1962; Martin, Wegner, 1979) are mentioned.

According to this method, the entire waterflooding area is covered with a small number of streamtubes between each pair of interacting injector and producer wells. Streamtube boundaries are determined by streamlines constructed from the solution of a steady state single-phase problem, and are considered unchanged during the entire waterflooding process. The main principle of this method, which makes it possible to speed up the solution of the two-phase flow problem by orders of magnitude, is to construct an analytical solution to the one-dimensional Buckley-Leverett problem to describe the saturation transfer along streamtubes with impermeable lateral boundaries. To determine the flow rate for each streamtube at each moment of time, its total resistance is calculated, which depends on the instantaneous internal distribution of saturation.

In (Higgins, Leighton, 1962; Martin, Wegner, 1979) it was shown that the assumption about the invariability of the shape of the streamtubes does not entail a significant error in the solution of the problem. So, for typical waterflooding schemes, the error in calculating oil recovery does not exceed 10%, when the ratio of the endpoint mobility of the water and oil phases lies in the range from 0.1 to 10. In these studies the flood cell was divided into 5 streamtubes.

Note that, in traditional approaches, the width of the streamtubes is a function of the longitudinal coordinate; the streamtubes can be heterogeneous in permeability and porosity, but they are one-dimensional. To describe the vertical heterogeneity of an oil reservoir, it is assumed to be layered, streamtubes of the same shape are built along each layer, and fluid exchange between the layers is not taken into account (for example, Higgins, Leighton, 1961; Yu.P. Borisov's method "VNII-1", Surguchev et al., 1984; Emanuel, Milliken, 1997). The distribution of well rates to flow rates along the streamtubes is carried out proportional to their resistance.

To take into account the streamtube method for two-dimensional flow in a heterogeneous cross-section of the reservoir, so-called hybrid scheme was proposed (Lake et al., 1981), which is a combination of the areal streamtube method and the traditional method of finite differences in the vertical section of the reservoir. It assumes that the areal flow is determined by the placement of wells, while the vertical behavior of the flow primarily depends on the geological structure of the reservoir and the method of oil displacement. The model of a typical cross-section of the reservoir between a pair of injection and production wells is described by a detailed grid with a step of the order of 0.1–1 m. From the solution of the two-phase flow problem using this model, the dependences for water fraction in the produced fluid on the number of injected pore volumes of water and the average saturation in the streamtube are obtained. Next, a solution to the areal problem is constructed by the streamtube method for a single-layer reservoir model. The pore volumes of fluid pumped into the streamtubes are used to determine their resistance and phase removal into the production well by scaling the response functions obtained from solving a two-dimensional problem in the cross section of the formation. This scheme has been generalized to simulate various methods of enhanced oil recovery (Lake et al., 1981; Emanuel et al., 1989; Renard, 1990). Later in the work (Hewett, Behrens, 1991), a detailed discussion of the scaling of properties was presented when determining the averaged response functions for inhomogeneous vertical sections and it was concluded that, in the general case, the inhomogeneity does not allow passing from two-dimensional problems to one-dimensional problems using scaling.

Another way to take into account the two-dimensional flow is demonstrated in (Baek, Hewett, 2000). Here, the cross section of each streamtube is again covered by a system of fixed streamtubes, along which a series of one-dimensional solutions of the Buckley-Leverett problem is constructed with recalculation of their resistance. The authors characterize the proposed method as a quick tool for assessing the impact of the uncertainty of the geological model, which, at the same time, is not a convenient method for solving problems of oil reservoir management.

An analysis of the further development of the streamtube method can be found, for example, in (Thiele, 1994; Al-Najem et al., 2012).

Present paper is devoted to the study of the flow model with a fixed streamtube, presented in (Mazo et al., 2017; Potashev et al., 2016; Mazo, Potashev, 2020). It differs from the above methods in that the streamtube is two-dimensional, reflecting the characteristic vertical section of the formation between the injection and production wells. In a given cross-section of variable width, corresponding to the width of the streamtube, a two-dimensional problem of two-phase flow is solved on a detailed computational grid. Thus, in this model, in contrast to the methods listed above, the simplifications necessary for constructing an analytical solution of the Buckley-Leverett problems along one-dimensional streamtubes are not introduced.

In this paper, we consider the question of the possibility of constructing only one – an effective streamtube, which would completely determine the interaction of a pair of injection and production wells. In this case, the width of the streamtube is assumed to be unchanged (fixed). The error in determining the performance of wells from the solution of the two-phase flow problem in the streamtube is estimated, and an algorithm is proposed for constructing an effective streamtube that minimizes this error.

An effective streamtube construction has been demonstrated for typical waterflooding patterns, described by the interaction of one pair of injector and producer: for a direct and inverted seven-spot patterns, equivalent to a regular four-spot pattern, and a staggered line drive, equivalent to a five-spot pattern (Willhite, 1986).

The obtained effective streamtube for given pattern can be used as a ready-made template for subsequent modeling of geological and technical treatments in the corresponding waterflooding cells.

When constructing streamtubes, streamlines are used that are plotted in the horizontal plane according to the vector field of Darcy velocity determined from the solution of a two-dimensional steady state flow problem. Such a model can be obtained by averaging a three-dimensional model of global waterflooding over the formation thickness at the time moment of a local event (Mazo et al., 2017).

With the aim of investigating the fundamental possibility and accuracy of reproducing the two-phase flow interaction of the injector and producer using a single streamtube of a fixed width, this work considers the case of a uniform oil reservoir of constant thickness, penetrated by vertical perfect wells, neglecting capillary and gravity effects, as well as the solid matrix and fluids compressibility. Under such conditions, "accurate" well performance indicators can be obtained by solving a two-dimensional problem of two-phase flow in the area of a waterflooding element on a detailed computational grid.

1. Two-dimensional problem of two-phase flow in a flooding element

Under mentioned assumptions, the process of waterflooding of a vertically averaged petroleum reservoir within the periodicity cell D of a given pattern of wells modeled by point sources will be described by a system of two-dimensional equations in the horizontal plane (x,y) in dimensionless variables (Barenblatt, 1984) for pressure:

$$t>0,\ (x,y)\in D:\ \nabla\cdot\mathbf{u}=\sum_{j=1}^{N_W}q_j\,\delta(x-x_j)\delta(y-y_j),$$
$$\mathbf{u}=-\varphi(s)\nabla p, \tag{1.1}$$

and for saturation:

$$t>0,\ (x,y)\in D:\ \frac{\partial s}{\partial t}+\nabla\cdot\left[f(s)\mathbf{u}\right]=$$
$$\sum_{j=1}^{N_W}f\left(s(x_j,y_j)\right)q_j\,\delta(x-x_j)\delta(y-y_j) \tag{1.2}$$

with initial condition expressing the absence of a water phase in the formation:

$$t=0,\ (x,y)\in D:\ s=0, \tag{1.3}$$

boundary condition reflecting the absence of fluid flow through the boundary ∂D of the flooding element due to the symmetry of the arrangement of equivalent wells:

$$t>0,\ (x,y)\in\partial D:\ \frac{\partial p}{\partial n}=0, \tag{1.4}$$

the given bottomhole pressure and the condition of complete saturation with the water phase on injection wells:

$$t>0,\ (x,y)=\left(x_j^I,y_j^I\right):\ p=1,\ \ s=1,\ \ j=1..N_W^I \tag{1.5}$$

and the given bottomhole pressure on production wells:

$$t>0,\ (x,y)=\left(x_j^P,y_j^P\right):\ p=0,\ \ j=1..N_W^P \tag{1.6}$$

Here t – time; p – pressure in fluid; s – water saturation; $\mathbf{u}$ – Darcy velocity of two phase fluid; $\varphi(s)=f_1(s)+K_\mu f_2(s)$ – function of total mobility of a two-phase mixture; $f(s)=f_1(s)/\varphi(s)$ – Buckley-Leverett function; $f_1(s)=s^3$, $f_2(s)=(1-s)^3$ – relative permeability functions for water and oil phases respectively; (x_j,y_j) – wells coordinates; $\delta(x)$ – Dirac delta function; q_j – specific per unit of formation thickness well rates determined when solving the problem; $N_W=N_W^P+N_W^I$ – number of wells in area D; upper indexes P,I refer to the parameters of production and injection wells respectively. Viscosity ratio $K_\mu=\mu_1/\mu_2$ was assumed to be equal to one.

The geometry of the area D and the coordinates (x_j,y_j) of the wells are set depending on the simulated waterflooding patterns (Figure 1).

Since each considered well pattern scheme is fully

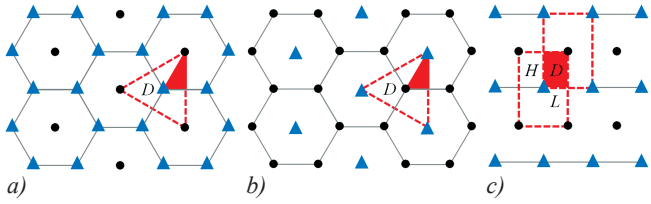

Fig. 1. Well placement schemes: a) seven-spot (analogous to regular inverse four-spot); b) inverted seven-spot (analogue of direct four-spot); c) staggered line drive (analogue of the direct (upper element) and inverse (lower) five-spot); filling indicates the area of cells of periodicity of flooding elements; • – producers; ▲ – injectors

described by the interaction of one pair of injector and producer ($N_W^P=N_W^I=1$), for simplicity we will denote the specific flow rate and water cut of the producer obtained from the solution of problem (1.1)–(1.6), without additional indices, hereinafter calling them "*exact*" or "*true*" values:

$$q(t)\equiv q_1^P(t),\quad F(t)\equiv f\left(s\left(\mathbf{x}_1^P,t\right)\right). \tag{1.7}$$

2. One-dimensional problem of two-phase flow in a streamtube

To construct fixed streamtubes during the entire waterflooding process, a series of N streamlines are used, constructed from the vector field of the Darcy velocity $\mathbf{u}(x,y)$, found from the solution of the steady state single-phase flow problem (1.1), (1.4) in the region D with the initial saturation distribution (1.3).

Streamlines are determined as lines tangent to which at each point is directed along a vector $\mathbf{u}$. Each streamline has its own length $\lambda_i (i=1\ldots N)$ and, according to the impermeability condition of the region boundary ∂D (1.4), begins (at $l=0$) in the injector and ends (at $l=\lambda_i$) in the producer.

We will understand by the streamtube the area of the reservoir bounded above and below by its top and bottom surfaces, and on the sides by vertical surfaces passing along any two streamlines. The assumption of a weak variability of the streamline form along the vertical near wells that are imperfect in the degree of penetration of the stratified formation is performed in a wide range of typical values of the layers permeability (Spirina et al., 2019). The boundaries of the streamtube are impermeable, and the total flow rate through its normal sections takes the same value along the entire length. Along each streamline i, it is possible to construct a separate streamtube, which in what follows will be called the original streamtube (OST), which has a variable relative width $w_i(l)$ inversely proportional to the absolute Darcy velocity in a given point l of the streamline:

$$w_i(l)=1\big/\left|\mathbf{u}(l)\right|,\quad 0\le l\le\lambda_i,\quad i=1..N. \tag{2.1}$$

When constructing the OST, it is assumed that the Darcy velocity on the entire surface of wells with radius r_w is the same and is determined by their flow rate (Potashev, Akhunov, 2020). Therefore, the function of the relative width of all N OST takes the same values at their boundaries:

$$w_L\equiv w_i(0)=2\pi r_w\big/\left|q^I\right|,\ w_R\equiv w_i(\lambda_i)=2\pi r_w\big/\left|q^P\right|,$$
$$i=1..N. \tag{2.2}$$

The formulation of two-phase flow problems in each OST $i=1\ldots N$ with a constant function of the relative width $w_i(l)$ during flooding process will contain (Mazo et al., 2017) the equations for pressure and saturation:

$$t>0,\ 0<l<\lambda_i:\ \frac{\partial}{\partial l}(w_iu_i)=0,\quad u_i=-\varphi(s_i)\frac{\partial p_i}{\partial l}, \tag{2.3}$$

$$t>0,\ 0<l<\lambda_i:\ \frac{\partial s_i}{\partial t}+\frac{1}{w_i}\frac{\partial}{\partial l}\left[w_if(s_i)u_i\right]=0, \tag{2.4}$$

initial condition corresponding to the condition (1.3):

$$t=0,\quad 0\le l\le\lambda_i:\quad s_i=0, \tag{2.5}$$

and boundary conditions corresponding to well conditions (1.5), (1.6):

$$t>0,\quad l=0:\ p_i=1,\ s_i=1;$$
$$t>0,\quad l=\lambda_i:\ p_i=0. \tag{2.6}$$

The solutions of problems (2.3)–(2.6) makes it possible to find functions of pressure $p_i(t,l)$, saturation $s_i(t,l)$ and Darcy velocity $u_i(t,l)$ for each ITT ($i=1\ldots N$).

Since all the original streamtubes, in accordance with the distribution in the region D of the streamlines, are characterized by different lengths λ_i and different functions of the relative width $w_i(l)$, the instantaneous position of the saturation front in each OST and the phase composition of the fluid flow from it to the producer are different.

The specific production rate of a producer and its water cut at a time t can be approximately calculated from the solutions in the OSTs using the formulas:

$$q(t)=\int_0^{2\pi}u^n(t,\varphi)r_wd\varphi\approx 2\pi r_w\sum_{i=1}^N\alpha_iu_i(t,\lambda_i), \tag{2.7}$$

$$F(t)=\frac{1}{q(t)}\int_0^{2\pi}f\left(s(t,\varphi)\right)u^n(t,\varphi)r_wd\varphi\approx$$
$$\sum_{i=1}^N\alpha_if\left(s_i(t,\lambda_i)\right)u_i(t,\lambda_i)\Big/\sum_{i=1}^N\alpha_iu_i(t,\lambda_i), \tag{2.8}$$

where α_i is the fraction of the transverse contour of the well, which accounts for the inflow from the side of the OST i, while:

$$\sum_{i=1}^N\alpha_i=1. \tag{2.9}$$

The purpose of this work is to develop a method for replacing all the OSTs with one fixed effective streamtube (EST), the solution of the problem of two-phase flow along which would allow determining the well performance with a minimum error.

3. Effective streamtube construction

The construction of an effective streamtube from a set of original streamtubes will be performed by normalizing

all OSTs along their length, followed by calculating the weighted average function of the relative width w over the given weight coefficients. Weights can be understood as coefficients α_i. Then the problem of constructing an EST is reduced to finding the optimal values α_i.

All parameters and characteristics of the EST will be denoted similarly to the parameters of the OSTs without a subscript.

Normalizing all OSTs by length will lead to a new longitudinal coordinate:

$$x = \frac{l}{\lambda_i} \in [0,1].$$

The relative width $w(x)$ of the EST in each section x will be calculated as a linear combination of the relative widths $w_i(x)$ of all OSTs with the coefficients α_i:

$$w(x) = \sum_{i=1}^{N} \alpha_i w_i(x). \tag{3.1}$$

Based on the statement (2.3)–(2.6) in OST, and introducing the notations:

$$\bar{u} = \lambda u, \quad \bar{t} = \lambda^{-2} t, \quad \lambda = \sum_{i=1}^{N} \alpha_i \lambda_i, \tag{3.2}$$

we write the statement of the two-phase flow problem in an EST:

$$\bar{t} > 0,\ 0 < x < 1: \begin{cases} \dfrac{\partial}{\partial x}(w\bar{u}) = 0, \quad \bar{u} = -\varphi(s)\dfrac{\partial p}{\partial x}, \\ \dfrac{\partial s}{\partial \bar{t}} + \dfrac{1}{w}\dfrac{\partial}{\partial x}\left[f(s) w \bar{u}\right] = 0; \end{cases} \tag{3.3}$$

$$\begin{aligned} &\bar{t} = 0, \quad 0 \le x \le 1: \quad s = 0; \\ &\bar{t} > 0, \quad x = 0: \quad p = 1, \quad s = 1; \\ &\bar{t} > 0, \quad x = 1: \quad p = 0. \end{aligned} \tag{3.4}$$

Calculating the whole inflow of fluid to the producer using the solution of problem (3.3)–(3.4) in one EST, according to (2.7), (2.8) and normalization (3.2), we obtain formulas for calculating the specific flow rate and water cut of the well according to fixed streamtube model in EST (subscript "E"):

$$q_E(t) = 2\pi r_w \bar{u}\left(\lambda^2 \bar{t}, 1\right) / \lambda, \tag{3.5}$$

$$F_E(t) = f\left(s\left(\lambda^2 \bar{t}, 1\right)\right). \tag{3.6}$$

Note that the transition from a set of OST with true properties of an oil reservoir to an averaged EST calls for the implementation of upscaling procedures (Christie, 1996) – determining the effective values of absolute permeability and functions of relative permeabilities in EST that would guaranteed the minimum error in calculating the total and phase flows through the normal sections of the EST. Nevertheless, leaving the solution of this issue for the future, we write equations (3.3) and (3.6) under the assumption that the absolute and relative phase permeabilities in the EST remain initial. Thus, an increase in the accuracy of approximation (1.7) of the true performance indicators $q(t)$, $F(t)$ of the producer by the values calculated from the solution of the problem in EST using equations (3.5), (3.6) will be ensured by determining the weight coefficients α_i that minimize the functional of the mean deviation:

$$R(\alpha_i) = \frac{1}{2}\left(R_q(\alpha_i) + R_F(\alpha_i)\right), \tag{3.7}$$

where R_q, R_F are the maximum deviations of the values $q(t)$, $F(t)$ от $q_E(t)$, $F_E(t)$ for the time period T of calculating the dynamics of the specific flow rate and water cut.

The number of the sought weight coefficients α_i that minimize functional (3.7) is equal to the number N of OST. Assuming that the degree of influence α_i of a separate OST on the flow rate and water cut of the well decreases linearly with an increase in the streamtube length λ_i, and taking into account condition (2.9), we can write the dependence:

$$\alpha_i = \alpha_0 - \alpha \lambda_i, \quad \alpha_0 = \frac{1}{N}\left(1 + \alpha \sum_{i=1}^{N} \lambda_i\right), \quad \alpha \ge 0. \tag{3.8}$$

Thus, the problem of finding the optimal values of the coefficients α_i is reduced to the problem of one-dimensional minimization with finding only one parameter:

$$\alpha^* = \arg\left[\min_{\alpha \ge 0} R(\alpha)\right]. \tag{3.9}$$

Note that the shape of such EST in general will not coincide with the shape of the streamtubes bounded by any streamlines.

4. Results

As examples of constructing EST, three typical flooding cells of periodicity were considered (Figure 1), corresponding to seven different flooding patterns: direct and inverted four-, five- and seven-spot, as well as staggered line drive (Willhite, 1986).

To determine the "exact" values of the flow rate and water cut (1.7), a numerical solution of problem (1.1)–(1.6) was constructed according to the IMPES scheme (implicit pressure explicit saturation) (Aziz, Settari, 1979). In accordance with the study of the convergence of the solution of the two-phase flow problem for each flooding pattern, computational grids were used containing from 200 to 400 finite elements between the injection and production wells, which made it possible to reproduce in detail the frontal character of saturation propagation. The numerical solution of problem (3.3)–(3.4) in an EST was also performed using the IMPES scheme. The detail degree of the computational grid in EST corresponded to the detail degree of two-dimensional grids. The one-dimensional minimization problem (3.9) was solved by the golden section method. The calculation of the deviation R according to the formulas (3.7) for each flooding pattern was carried out until the moment of time T corresponding to the achievement of the water cut value F=0.98.

4.1. Seven-spot pattern

The cell of the periodicity of the seven-spot pattern flooding element is similar to the cell of the inverted four-spot pattern (Figure 1, a), when an injector is located in the center of the triangular area D, and producers are located at its tops. The distance between the producers was set L=1, the radius of the wells was set equal r_w=0.001. The structure of streamlines plotted inside the region D and the distribution of saturation in it at different time moments are shown in Figure 2.

The EST was constructed for sets containing a different number N of OSTs, uniformly covering the periodicity cell of the flooding element D (Figure 1, a). The lengths λ_i of the OSTs constructed in this way took values from 0.576 to 0.776. The function of the relative width $w(x)$ of the EST (Figure 3, b) and its reduced length λ=0.62 turned out to be weakly dependent on the number of OSTs, despite the fact that the initial functions of all OSTs significantly differed from each other (Figure 3, a). The stability of the function of the relative width $w(x)$ is achieved by combinations of coefficients α_i that determine the contribution of each original streamtube to the effective one. For example, at N=5 the minimum and maximum values of the coefficients α_i were 0.066 and 0.26, differing by about 4 times, and at N=120 they were equal to 0.0064 and 0.0088, differing by less than 1.5 times.

The form of the resulting function $w(x)$ for EST made it possible to propose its simplified piecewise linear approximation $W(x)\approx w(x)/w_R$, which is more convenient for fast engineering calculations. Taking into account the notation (2.2), we write the function $W(x)$ in the form:

$$W(x)=\begin{cases}\omega+ax, & 0\le x<\xi,\\ 1+b(1-x), & \xi\le x<1.\end{cases} \qquad (4.1)$$

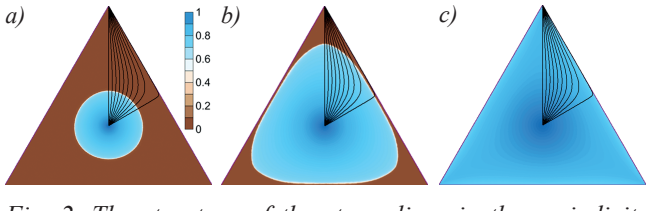

Fig. 2. The structure of the streamlines in the periodicity cell of the flooding element of the seven-spot pattern and saturation distribution at times a) t=0.1, b) t=0.4, c) t=3.0

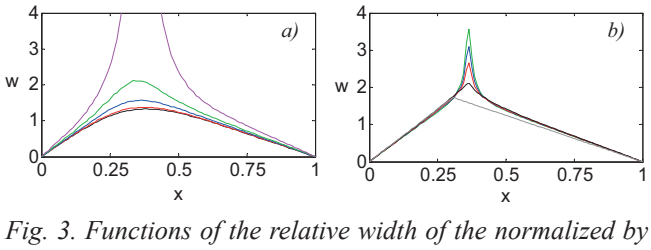

Fig. 3. Functions of the relative width of the normalized by the length streamtubes: a) for all OST at N = 5; b) for EST at N = 3, 5, 10, 120 and their approximation by (4.1) (double thickness line)

Here $\omega=w_L/w_R$; and the coefficients a, b reflect the convergent and divergent behavior of the flow near the injector and producer, respectively, and are determined by the derivatives of the function $w(x)$ at the boundaries: $a=w'(0)/w_R$, $b=-w'(1)/w_R$. The parameter $\xi=(b+1-\omega)/(a+b)$ is defined as the intersection point of both straight lines (4.1).

When constructing the function $W(x)$ by formula (4.1) (Figure 3, b), the following values of its parameters were determined ω=2, $\alpha=a/(\lambda/r_w)$=2.052, $\beta=b/(\lambda/r_w)$=0.906.

In Figure 4 it is shown a comparison of the exact dynamics of the specific flow rate and water cut of a producer with the solutions of the problem in various streamtubes. The use of effective streamtube or its approximation (4.1) (Figure 4, b) leads to a significant increase in the accuracy of reproduction of functions $q(t)$, $F(t)$ in comparison with the use of any of the original streamtubes (Figure 4, b) (Table 1).

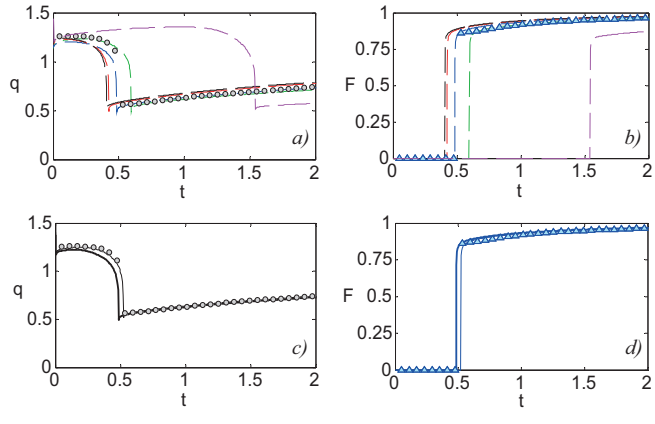

Fig. 4. Comparison of true (marker) and calculated in streamtubes (lines) flow rate and water cut: a), b) – from the solution of problem (2.3)–(2.6) in each ITT at N=5; c), d) – from the solution of problem (3.3)–(3.4) in ETT using the true function w (thin lines) and its approximation W (lines of double thickness)

Parameter	OST-1	OST-2	OST-3	OST-4	OST-5	EST	$W(x)$
R_q	0.118	0.106	0.081	0.077	0.694	0.027	0.072
R_F	0.114	0.095	0.034	0.079	0.578	0.017	0.034
R	**0.116**	**0.101**	**0.058**	**0.078**	**0.636**	**0.022**	**0.053**

Table 1. The error in calculating the flow rate and water cut for the original and effective streamtubes for a seven-spot pattern at N=5

4.2. Inverted seven-spot pattern

The periodicity cell of the flooding element of the inverted seven-spot pattern is similar to the cell of the regular four-spot pattern (Figure 1, b), when a production well is located in the center of the triangular area D, and injection wells are located at its tops. The distance between the injectors and the well radius were also set equal to L=1 and r_w=0.001.

The structure of the streamlines built under the initial conditions (Figure 5) coincides with the structure of

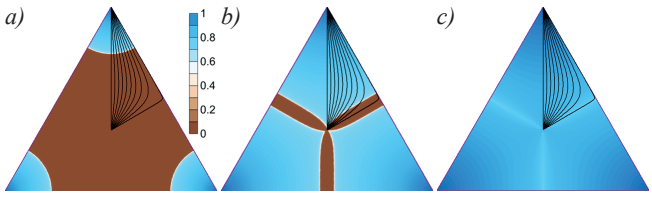

Fig. 5. The structure of streamlines in the periodicity cell of the flooding element of the inverted seven-spot pattern and the saturation distribution at times a) t=0.1, b) t=0.5, c) t=3.0

the streamlines in the case of a seven-spot well spacing scheme in Section 4.1.

The general features of constructing an EST of a inverted seven-spot pattern with a different number of OST remained similar to the case of a standard seven-spot pattern (Section 4.1). The function of the relative width $w(x)$ of the EST and its approximation $W(x)$ in the form (4.1) with parameters ω=0.5, α=0.492, β=0.952, λ=0.61 are shown in Figure 6, a.

The Figure 6, b shows a comparison of the true dynamics of production rate and water cut of a producer with the results of solving the problem in EST. In contrast to the standard seven-spot scheme, the deviation of the water cut functions $F(t)$ and $F_E(t)$ turns out to be more significant, which is explained by the water flooding structure. Areas of slow oil displacement are formed near the production well (Figure 5, b), as a result of which the increase in water cut also slows down compared to the standard seven-spot pattern. In this example, the previously made remark about the need to upscale the relative phase permeability functions $f_1(s)$, $f_2(s)$ for the problem along one streamtube becomes clearer.

The use of an EST or its approximation (4.1), in comparison with the use of arbitrary OST, reduces the error in the reproduction of functions $q(t)$, $F(t)$ at N=5, on average, by a factor of 2.5 (Table 2).

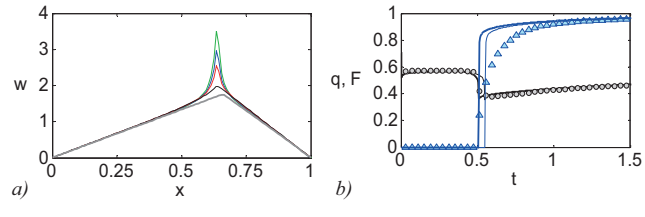

Fig. 6. Results of constructing an EST for an inverted seven-spot pattern: a) the relative width w of the EST at N=3, 5, 10, 120 (thin lines) and approximation W (double thickness line); b) production rate (black) and water cut (blue) – true values (markers) and calculated from ETT with the true relative width w (thin lines) and with the approximated width W (double thickness lines)

Parameter	OST-1	OST-2	OST-3	OST-4	OST-5	EST	$W(x)$
R_q	0.083	0.069	0.033	0.125	0.217	0.037	0.021
R_F	0.149	0.135	0.091	0.118	0.424	0.078	0.099
R	**0.116**	**0.102**	**0.062**	**0.122**	**0.321**	**0.058**	**0.060**

Table 2. The error in calculating the flow rate and water cut for the original and effective streamtubes for an inverted seven-spot pattern at N=5

4.3. Staggered line drive pattern

Rectangular periodicity cells D of the flooding element of the staggered line drive pattern are similar to the cells of the standard and inverted five-spot patterns (Figure 1, c). Let's set the width of the area D as L=1. The height H for the case of equidistant wells coincides with L, and in a more general case can take arbitrary values. In the lower left corner we will place the injector, in the upper right corner – the producer. The radius of the wells is set equal r_w=0.001.

For the case H=L, the solution of problem (1.1)–(1.6) in the area D is symmetric with respect to the diagonal extending from the injector to the producer (Figure 7). At other values θ=L/H of the ratio, this symmetry is violated (Figure 8).

All calculations with the construction of EST were carried out for various ratios θ={1/2, 2/3, 3/4, 4/5, 1/1}. According to the form of the obtained EST and taking into account that w_L=w_R, for all considered cases the following form of approximation was used (Figure 9):

$$\frac{w(x)}{w_R} \approx W(x) = \begin{cases} 1 + x(h-1)/d, & 0 \le x \le d; \\ h, & d < x < 1-d; \\ 1 + (1-x)(h-1)/d, & 1-d \le x \le 1. \end{cases} \quad (4.2)$$

Coefficients d, h of the dependence (4.2) were found for each ratio θ from the condition of the best approximation of the constructed EST. The values R, R_q, R_F are given in Table 3.

To construct the EST based on the approximate relationship (4.2) with an arbitrary aspect ratio of the sides of the flooding element D, the coefficients d, h were approximated by functions of the form factor θ (Figure 10, a, b):

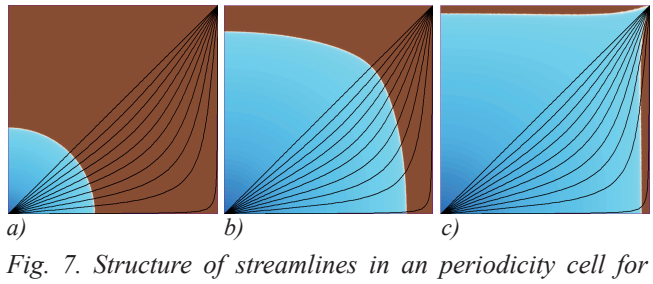

Fig. 7. Structure of streamlines in an periodicity cell for the staggered line drive pattern at H=L and saturation distribution at times a) t=0.5, b) t=2.5, c) t=4

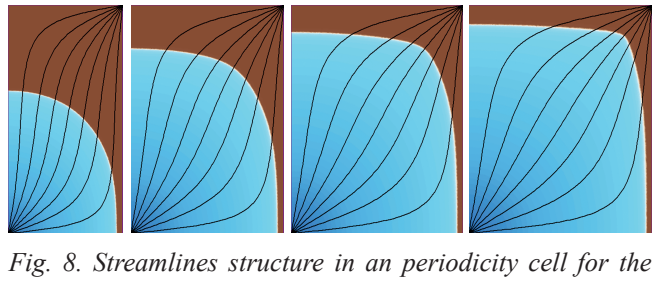

Fig. 8. Streamlines structure in an periodicity cell for the staggered line drive pattern at θ=1/2, 2/3, 3/4, 4/5 (left to right) and saturation distribution at time moment t=4

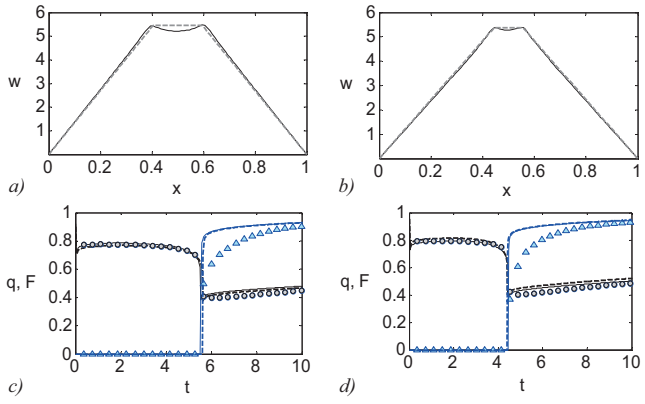

Fig. 9. Results of EST construction for staggered line drive pattern at θ=2/3 (a, c) and θ=4/5 (b, d): a), b) – relative width w (solid line) and approximation W according to (4.2) (dashed line); c, d) production rate (black) and water cut (blue) – true values (markers) and calculated from EST with the true relative width w (solid lines) and with the approximated width W (dashed lines)

L / H	1 / 2	2 / 3	3 / 4	4 / 5	1 / 1
R_q	0.058	0.042	0.052	0.052	0.044
R_F	0.143	0.133	0.159	0.157	0.169
R	**0.100**	**0.087**	**0.105**	**0.104**	**0.106**

Table 3. The error in calculating the flow rate and water cut using the effective streamtube for staggered line drive pattern

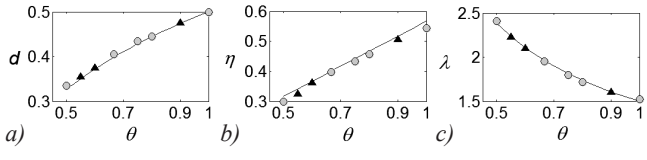

Fig. 10. Dependences of the coefficients d (a), η (b), λ (c) approximation (4.2) of the relative width of the EST of the staggered line drive pattern on the aspect ratio of the flooding element: solid line – approximate dependences (4.3), (4.4); markers – approximation (•) and forecast (▲) points

$$d = \frac{1}{4}\ln\theta + \frac{1}{2}; \quad \eta \equiv h/(\lambda/r_w) = \frac{7}{20}\theta + \frac{7}{13}. \qquad (4.3)$$

The dependence of the reduced EST length λ in (4.3) on the form factor θ was approximated by the function (Figure 10, c):

$$\lambda = \frac{3}{2}L\theta^{-\frac{2}{3}}. \qquad (4.4)$$

Testing of the proposed approximations (4.3), (4.4) for three additional values of the form factor θ={0.55,0.6,0.9} has shown good agreement between the approximated and true values of the coefficients d, η, λ. The average forecast error of the coefficient d was 0.8%, of the coefficient η was 2.9%, and of the value λ was 0.27% (Figure 10).

5. Conclusion

A method is proposed for constructing a single effective fixed streamtube that describes the interaction of a pair of injection and production wells, which makes it possible to reduce the dimension of the problem of the two-phase flow in petroleum reservoir without significant loss of accuracy in calculating well performance.

It is shown that for typical waterflooding schemes of a homogeneous reservoir, the shape of the effective streamtube can be approximated by piecewise linear functions. For a staggered line drive flooding pattern, similar to five-spot pattern, simple functional dependencies are proposed that make it possible to build an effective streamtube with an arbitrary ratio of the sides of the flooding element.

The constructed streamtubes can be used as ready-made templates for the subsequent modeling of complex geological and technical treatments in the corresponding flooding elements with a decrease in the dimension of the problem.

The described algorithm is demonstrated by the example of waterflooding of an incompressible homogeneous reservoir of constant thickness without taking into account capillary and gravity effects, but it can be applied in a more general case without the indicated restrictions.

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